

Newbattle Community High School
Higher Mathematics

Key Facts Q&A

Ways of using this booklet:

- 1) Write the questions on cards with the answers on the back and test yourself
- 2) Work with a friend who is also doing Higher maths to take turns reading a random question and answering
- 3) Ask a friend or family member** to test you by reading questions (on the left-hand side) to you

The questions are on the left-hand side of each page and the answers are on the right.

**If the person who is testing you has not done Higher maths before, some maths symbols have been written out phonetically to help them read them out loud. Some formulae in questions are printed large. These can be held up or shown.

All Units: Equations and Graphs

How do you find the y intercepts of a graph?	Substitute $x = 0$ into its equation
How do you find the roots of a graph?	Substitute $y = 0$ into its equation and solve
What three points do you need to indicate on the sketch of any graph? (if they exist)	<ol style="list-style-type: none"> 1. Stationary points 2. Roots 3. y intercept
What does \mathbb{Z} mean?	Integers
What does \mathbb{R} mean?	Real numbers

All Units: Trigonometric Graphs and Equations

For what three values of x between 0 and 360 is $\sin x$ zero?	0° , 180° and 360°	
For what two values of x between 0 and 360 is $\cos x$ zero?	90° and 270°	
For what three values of x between 0 and 360 is $\tan x$ zero?	0° , 180° and 360°	
What is $\sin 30^\circ$ <u>(sine thirty degrees)</u>	$\frac{1}{2}$	
What is $\cos 30^\circ$ <u>(cos thirty degrees)</u>	$\frac{\sqrt{3}}{2}$	<u>(root three over two)</u>
What is $\tan 30^\circ$ <u>(tan thirty degrees)</u>	$\frac{1}{\sqrt{3}}$	<u>(one over root three)</u>
What is $\sin 45^\circ$ <u>(sine forty five degrees)</u>	$\frac{1}{\sqrt{2}}$	<u>(one over root two)</u>
What is $\cos 45^\circ$ <u>(cos forty five degrees)</u>	$\frac{1}{\sqrt{2}}$	<u>(one over root two)</u>
What is $\tan 45^\circ$ <u>(tan forty five degrees)</u>	1	
What is $\cos 60^\circ$ <u>(cos sixty degrees)</u>	$\frac{1}{2}$	

What is $\sin 60^\circ$? <u>(sine sixty degrees)</u>	$\frac{\sqrt{3}}{2}$ <u>(root three over two)</u>
What is $\tan 60^\circ$? <u>(tan sixty degrees)</u>	$\sqrt{3}$ <u>(root three)</u>
What is $\cos 0^\circ$? <u>(cos zero degrees)</u>	1
What is $\sin 90^\circ$? <u>(sine ninety degrees)</u>	1
What is $\cos 180^\circ$? <u>(cos one eighty degrees)</u>	-1
What is 180 degrees in radians? <u>(Pi)</u>	π
What is 360 degrees in radians? <u>(Two Pi)</u>	2π
What is 90 degrees in radians? <u>(Pi over two)</u>	$\frac{\pi}{2}$
What is π radians in degrees?	180°
What is 2π radians in degrees?	360°
What is $\frac{\pi}{2}$ radians in degrees?	90°
What is the formula for tan? <u>(sine over cos)</u>	$\frac{\sin x}{\cos x}$
How do you change from degrees to radians?	Divide by 180 and multiply by Pi
How is the range of $\sin x$ and $\cos x$ restricted?	It is between -1 and 1
How do you find the amplitude of a trigonometric function from its equation?	It's the number in front of sin, cos or tan
How do you find the frequency of a trigonometric function from its equation?	It's the number in front of x

Unit 1 Outcome 4: Recurrence Relations

For a limit to exist, the value of a must be...	between -1 and 1
What is the formula for the limit of a recurrence relation?	$L = \frac{b}{1-a}$ <u>(L equals b over 1 minus a)</u>
What sentence do you HAVE to write for communication marks when you are finding a limit?	A limit exists because ____ is between -1 and 1

Unit 1 Outcome 1: The Straight Line

What is the formula for the gradient between two points?	$m = \frac{y_2 - y_1}{x_2 - x_1}$ <p style="text-align: center;"><u>(y two minus y one over x two minus x one)</u></p>
What is the general formula for the equation of a straight line?	$y - b = m(x - a)$ <p style="text-align: center;"><u>y minus b equals m x minus a</u></p>
How do you find the gradient of a straight line if you only know its equation?	<ol style="list-style-type: none"> 1. Rearrange to make y the subject 2. The gradient is the coefficient of x
What formula do you use to find the size of the angle between a straight line and the positive direction of the x-axis?	$m = \tan \theta$ <p style="text-align: center;"><u>m equals tan theta</u></p>
What is the rule for the gradient of parallel lines?	They are the same
What two things do you to do find a perpendicular gradient?	<ol style="list-style-type: none"> 1. Turn it upside down 2. Change the sign
What is the distance formula?	$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ <p style="text-align: center;"><u>Square root of x two minus x one squared add y two minus y one squared</u></p>
What are the two properties of the perpendicular bisector of a line?	<ol style="list-style-type: none"> 1. It goes through the midpoint of the line 2. It is perpendicular to the line
What are the two properties of the median of a triangle through point A?	<ol style="list-style-type: none"> 1. It goes through point A 2. It goes through the midpoint of the opposite side
What are the two properties of the altitude of a triangle through point A?	<ol style="list-style-type: none"> 1. It goes through point A 2. It is perpendicular to the opposite side
How do you find the gradient of an altitude of a triangle through point A?	<ol style="list-style-type: none"> 1. Find the gradient of the side opposite A 2. Flip and change sign
How do you find the gradient of the median of a triangle through point A?	<ol style="list-style-type: none"> 1. Find the midpoint of the side opposite A 2. Find the gradient from the midpoint to A

Unit 1 Outcome 3: Differentiation

What are the two basic steps for differentiating powers?	1. Multiply by the old power 2. Take one away from the power
How do you find the derived function ?	Differentiate
How do you find the derivative ?	Differentiate
How do you find a rate of change ?	Differentiate
How do we find the gradient of a tangent at a particular value of x ?	Differentiate and substitute in x
What do we know about $\frac{dy}{dx}$ (<u>dee y by dee x</u>) if a function is increasing ?	It is greater than zero
What do we know about $\frac{dy}{dx}$ (<u>dee y by dee x</u>) if a function is decreasing ?	It is less than zero
What do we know about $\frac{dy}{dx}$ (<u>dee y by dee x</u>) if a function is stationary ?	It is equal to zero
What are the four types of stationary point?	1. Maximum 3. Rising point of inflection 2. Minimum 4. Falling point of inflection
If you are trying to find a maximum or minimum something in a particular interval, what three points do you check?	1. Stationary points 2. The start value 3. The end value
What sentence do you HAVE to write for communication marks in an exam in a stationary points question?	A stationary point exists when $\frac{dy}{dx} = 0$ (or $f'(x) = 0$)
When is a function not differentiable?	If it is not defined at a point

(ignore these questions if you were taught to use nature tables instead of the second derivative)

If $\frac{d^2y}{dx^2}$ (<u>dee two y by dee x squared</u>) is positive, what does this tell us about a stationary point	It is a minimum
If $\frac{d^2y}{dx^2}$ (<u>dee two y by dee x squared</u>) is negative, what does this tell us about a stationary point	It is a maximum

Unit 1 Outcome 2: Functions and Related Graphs

What is the domain of a function?	What goes INTO the function
What is the range of a function?	What comes OUT of the function
What is the turning point of the quadratic function $a(x + p)^2 + q$? <u>a x plus p squared plus q</u>	$(-p, q)$
If a function contains a square root, its domain is restricted. The expression under the square root is...	≥ 0 <u>(greater than or equal to zero)</u>
If a function includes dividing, its domain is restricted. The expressions in the denominator are...	$\neq 0$ <u>(not equal to zero)</u>
How can you find the minimum or maximum value of a quadratic function?	Complete the square and look at the constant term
How has the graph of $y = f(x) + k$ been transformed? <u>y equals f of x, plus k</u>	It has been moved up by k units (or down if k is negative)
How has the graph of $y = f(x + k)$ been transformed? <u>y equals f of x plus k</u>	It has been moved to the left by k units (or to the right if k is negative)
How has the graph of $y = kf(x)$ been transformed? <u>y equals k f of x</u>	It has been stretched vertically by scale factor k
How has the graph of $y = f(kx)$ been transformed? <u>y equals f of k x</u>	It has been compressed horizontally by scale factor k
How has the graph of $y = -f(x)$ been transformed? <u>y equals minus f of x</u>	It has been reflected in the x axis <i>Alternative answer: it has been reflected upside down</i>
How has the graph of $y = f(-x)$ been transformed? <u>y equals f of minus x</u>	It has been reflected in the y axis <i>Alternative answer: it has been reflected left to right</i>

Unit 2 Outcome 1a: Quadratic Functions

What is the quadratic formula?	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
How can we find the minimum/maximum of a quadratic graph from its equation?	Complete the square
What is formula for the discriminant?	$b^2 - 4ac$ (b squared minus 4 a c)
If an equation has real and equal roots , what do we know about the discriminant?	It is equal to zero ($= 0$)
If an equation has real and distinct roots , what do we know about the discriminant?	It is greater than zero (> 0)
If an equation has real roots , what do we know about the discriminant?	It is greater than or equal to zero (≥ 0)
If an equation has no real roots , what do we know about the discriminant?	It is less than zero (< 0)
How can we tell if an equation has rational roots?	The discriminant is a perfect square
How can we tell if an equation has irrational roots?	The discriminant is not a perfect square
How do we solve a quadratic inequality?	Sketch the parabola and use the sketch to write down the values of x for which the graph is above or below the axes
How do we show that a line is tangent to a parabola	Show they only have one point of intersection <i>Alternative answer: show the discriminant of the equation of intersection is zero</i>

Unit 2 Outcome 1b: Polynomials

How do you find the remainder when a polynomial is divided by $x - a$? (x minus a)	Use synthetic division (with a)
When doing synthetic division, what do we call the number in the box at the end?	The remainder
When factorising a polynomial, what do you HAVE to write for a communication mark after synthetic division?	The remainder is zero, so _____ is a factor

When doing synthetic division, what do we call the coefficients along the bottom row?	The quotient
If a function has a repeated root, what does the graph look like at that point?	It is tangent to the x axis at that root
How do you show a function f has a root between a and b ?	Work out the function at a and b . Show that one is negative and the other positive

Unit 2 Outcome 2: Integration

What are the three steps for basic integration?	<ol style="list-style-type: none"> 1. Add one to the power, 2. Divide by the new power 3. Add C (or “add a constant”)
If we know $\frac{dy}{dx}$ (<u>dee y by dee x</u>), how do we find the original equation?	<ol style="list-style-type: none"> 1. Integrate 2. Substitute in a point on the curve to find C
How do we find the area between two curves?	Integral of top take away bottom
What must we remember for indefinite integrals?	$+ C$
How do you find the area underneath a curve?	<ol style="list-style-type: none"> 1. Integrate the equation of the curve 2. Substitute in the limits
What do you always need when the question asks you to find an area?	A sketch of the graphs to check if the area is above the axis, below the axis or split
What do we have to remember when finding an area below the x -axis?	The answer will be negative, and we have to deal with this appropriately
What are the three steps to remember when the area is partly above and partly below the x -axis?	<ol style="list-style-type: none"> 1. Calculate out the areas above and below the x-axis separately 2. Deal with negative signs 3. Add

Unit 2 Outcome 3: Compound Angle Formulae

What should you always check after finishing a trig equation?	Should the final answer be in degrees or radians?
What is the formula for sine in a right-angled triangle?	Opposite over Hypotenuse
What is the formula for cos in a right-angled triangle?	Adjacent over Hypotenuse
What is the formula for tan in a right-angled triangle?	Opposite over Adjacent

If you know solutions to a trig equation between 0 and 360, how can you find other solutions?	Add or take away multiples of 360
What are the three steps to solve an equation containing $\cos^2 x$ or $\sin^2 x$? (<u>cos squared x or sine squared x</u>)	<ol style="list-style-type: none"> 1. Rearrange to get zero on the right-hand side 2. Factorise it 3. Use each bracket to make a new equation

Unit 2 Outcome 4: The Circle

What does it mean if two circles are congruent ?	They have the same radius
What does it mean if two circles are concentric ?	They have the same centre
If you are asked to write down the equation of a circle, which one should you use?	The one with brackets in
What is the condition for whether or not a circle equation actually represents a circle?	The radius must be positive
What are the three steps to find the coordinates where a line meets a circle?	<ol style="list-style-type: none"> 1. make y (or x) the subject of the straight line 2. substitute this into the circle equation 3. solve the resulting quadratic equation
How do you show that a line is a tangent to a circle?	Show there is only one point of intersection <i>Alternative answer: show the discriminant of the equation of intersection is zero</i>
How do you show that a line and a circle do not intersect	Show there are no points of intersection <i>Alternative answer: show the discriminant of the equation of intersection is negative</i>
How do you find the gradient of a tangent to a circle?	It is perpendicular to the radius
What is a common tangent?	A line which is a tangent to two circles
How do I show that two circles have two points of intersection?	Show that the distance between the two centres is less than the sum of the two radii
How do I show that two circles do not intersect?	Show that the distance between the two centres is greater than the sum of the two radii
How do I show that two circles touch in one point?	Show that the distance between the two centres is equal to the sum of the two radii

Unit 3 Outcome 1: Vectors

FORMULA SHEET

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta \quad \text{or} \quad \mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$$

What are the three steps for finding the magnitude of a vector ?	1. Square all the components 2. Add them, 3. Square root
What is the formula for the components of a vector from A to B?	$\mathbf{b} - \mathbf{a}$
How do you show that vectors \mathbf{u} and \mathbf{v} are parallel?	Show that $\mathbf{u} = k\mathbf{v}$ for some k
If \mathbf{u} is a vector, what route is described geometrically by the negative vector $-\mathbf{u}$?	It is going backwards along \mathbf{u}
If \mathbf{u} and \mathbf{v} are vectors, what route is described geometrically by $\mathbf{u} + \mathbf{v}$?	Following \mathbf{u} and then \mathbf{v} , when joined nose to tail
What does it mean if three points are collinear?	They all lie in a straight line
How do you show three points are collinear?	1. Show two vectors joining them are parallel 2. State that they have a common point
What two things do you HAVE to state for a communication mark when showing points are collinear	1. That the vectors are parallel 2. And that they contain a common point
What is the condition for two vectors to be perpendicular?	Their dot product is zero
What formula do you need to find the angle between two vectors?	$\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \mathbf{b} \cos\theta$
When talking about the angle between two vectors, how should the vectors be positioned?	Either both pointing outwards Or both pointing inwards
How do you show that two vectors \mathbf{a} and \mathbf{b} are perpendicular?	Show that $\underline{\mathbf{a}} \cdot \underline{\mathbf{b}} = 0$ <u>(a dot b equals zero)</u>
What does it mean if a vector is a unit vector?	Its magnitude is 1
If you are told a vector, what are the two steps for finding a parallel unit vector ?	1. Calculate the magnitude 2. Divide through by the magnitude
What is $\underline{\mathbf{a}} \cdot \underline{\mathbf{a}}$ equal to? <u>(a dot a)</u>	$ \mathbf{a} ^2$ <u>(the magnitude of a squared)</u>
How do you expand the brackets in $\underline{\mathbf{a}} \cdot (\underline{\mathbf{b}} + \underline{\mathbf{c}})$? <u>(a dot brackets b plus c)</u>	$\underline{\mathbf{a}} \cdot \underline{\mathbf{b}} + \underline{\mathbf{a}} \cdot \underline{\mathbf{c}}$ <u>(a dot b plus a dot c)</u>

Unit 3 Outcome 3: Exponential and Logarithmic Functions

What is e correct to two decimal places?	2.72
In an exponential equation Ae^{kt} , what is A ?	The starting value
In an exponential equation Ae^{kt} , what is k ?	The rate of change
What two letter abbreviation do we use for the logarithm to the base e ?	LN
What is the inverse function of $y = a^x$? <u>y equals a to the power of x</u>	$y = \log_a x$ <u>y equals log to the base a of x</u>
What is the inverse function of $y = \log_a x$? <u>y equals log to the base a of x</u>	$y = a^x$ <u>y equals a to the power of x</u>
What is the inverse function of $y = e^x$? <u>y equals e to the power of x</u>	$y = \ln x$
What is the inverse function of $y = \ln x$? <u>y equals LN x</u>	$y = e^x$ <u>y equals e to the power of x</u>
What is the value of $\log 1$ for any base?	0
Which two points will an exponential graph (base a) always pass through?	$(0,1)$ and $(1,a)$
Which two points will a logarithmic graph (base a) always pass through?	$(1,0)$ and $(a,1)$
How do you rewrite $y = \log_a x$ in power form? <u>(y equals log to the base a of x)</u>	$x = a^y$ <u>(x equals a to the power y)</u>
How do you solve an equation with x in the power? (eg $4^x = 10$)	Take logs of both sides then bring x down to the front
How do you solve an equation containing the number e ?	By taking ln (natural log) of both sides
Laws of logs: When you add two logs with the same base, what happens to the numbers?	Multiply them
Laws of logs: When you take away two logs with the same base, what happens to the numbers?	Divide them

Laws of logs: How do you deal with a power inside a log?	Bring the power down to the front
A straight line graph has $\log x$ or $\log y$ on the axes. What are the three steps to find its equation?	<ol style="list-style-type: none"> 1. Find the equation of the line using $y = mx + c$ 2. Replace x and y with whatever is on the axes 3. Use laws of logs to rearrange

Unit 3 Outcome 2: Further Calculus

What do you get if you differentiate sine?	cos
What do you get if you differentiate cos?	-sine
What do you get if you integrate sine?	-cos plus C
What do you get if you integrate cos?	sine plus C
What do you have to remember about angles when integrating or differentiating sine or cos ?	Angles must be in radians and <u>not</u> degrees
How do you differentiate a bracket $(ax + b)^n$? <u>(a x plus b to the power n)</u>	<p>Use the chain rule</p> <p>Alternative answer</p> <ol style="list-style-type: none"> 1. <i>Multiply by the old power</i> 2. <i>Take one off the power</i> 3. <i>Differentiate the bracket and multiply at the front by your answer</i>
How do you integrate a bracket $(ax + b)^n$? <u>(a x plus b to the power n)</u>	<ol style="list-style-type: none"> 1. Add one to the power 2. Divide by the new power 3. Differentiate the bracket and divide by your answer

Unit 3 Outcome 4: The Wave Function

In a wave function question, what is the formula for k ?	$\sqrt{a^2 + b^2}$ <p>(square root of a squared plus b squared)</p>
In a wave function question, what is the formula for a ?	$\tan a = \frac{k \sin a}{k \cos a}$ <p>(tan a equals k sine a over k cos a)</p>